Charged Lepton Flavour and CP Violations: Theoretical Impact of Present and Future Experiments

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We shortly review and emphasize how $\ell^j \to \ell^i \gamma$ experiments and the searches for lepton e.d.m. are constraining New Physics model building. They are pure signals of new phenomena around the TeV scale since the SM contributions are definitely negligible. It is quite remarkable that they also give effective tests of the LFV&CPV in seesaw couplings and in grand-unified theories. In particular, the limits on d_e nicely complement the proton decay bounds in selecting O(10) models.

1. Introduction

The title of this short review could well be "Lepton Flavour and CP Violations without Neutrinos for Neutrino Physicists" since the aim here is to emphasize the impact of the present and near future experiments on charged lepton transitions on (a) the flavour and CP pattern of the new lepton-like states in theories beyond the SM and (b) the neutrino mass models. Indeed, FCNC and CPV are generic problems for theories that postulate new states at the TeV scales which is typically the case in string inspired solutions to the hierarchy problem: squark and sleptons in supersymmetric models and Kaluza-Klein states in models with large compact dimensions. Basically, the problems arise because particles get mass from different, generically unrelated, mechanisms: Higgs couplings for the usual fermions, supersymmetry breaking or compactification for the new heavy states. Any misalignment in flavour space or in CP phases between the resulting mass matrices yields FCNC or CPV effective operators from radiative corrections involving the heavy states. The present experiments already put either strong limits on these misalignments or lower bounds above the TeV for the new state masses. These constraints open a new framework to inves-

tigate both the flavour problem and the structure of the new theories, in particular, in the lepton sector.

Let us first summarize the basic facts.
- Neutrino oscillations require large lepton flavour (as defined by the charged leptons) mixing in the effective neutrino mass matrix, but this does not necessarily imply large mixing in the charged sector.
- Leptogenesis is viable within the seesaw model for neutrino masses if the Yukawa couplings of the neutrinos to the Higgs (i.e., the Dirac masses) have sizeable CP phases.
- In the somewhat minimal framework where only the seesaw mechanism to generate neutrino masses is added to the Standard Model, the LFV&CPV effects in the charged lepton physics are extremely tiny because of the analogue of the GIM mechanism: they depend on factors of $\Delta m_\nu^2 / M_W^2 < 10^{-24}$.
- Therefore, if observed in the current or planned experiments, LFV in charged decays ($\mu \to e\,\gamma$, $\tau \to \mu\,\gamma$) or CPV electric dipole moments (d_e, d_μ) would be signals of New Physics beyond the SM and seesaw models around the TeV region (e.g., supersymmetry).
- Conversely, the present and future bounds on these LFV&CPV transitions constrain New Physics around the TeV region (e.g., supersym-

metry) and require a LFV&CPV inhibition mechanism in the corresponding theories.

- Some of these experiments provide already relevant constraints on radiative corrections from new LFV&CPV couplings in theories at unattainable scales: GUT's, seesaw, flavour models ...

We shall emphasize here the last point to demonstrate the importance of these experiments. Indeed they provide already restrictions on the masses and couplings of very heavy states in seesaw and/or GUT models from their virtual contributions in radiative corrections to the effective low energy parameters in any new theory in the TeV region (*e.g.,* slepton masses in supersymmetry). Future experiments will make these constraints even more impressive.

The phenomenological information gathered from LFV&CPV in the charged sector is quite complementary to those provided by measurements of neutrino oscillations or from the assumption of leptogenesis for the generation of the baryon asymmetry in the universe. This has been discussed in several papers and reviews at a more technical level [1] and, here, we shall rather illustrate this aspect by comparing the explicit predictions of simple basic models with the available set of experimental data.

The variety of these data are reviewed by M. Aoki in these Proceedings [2], which we refer to for more detailed description of different experiments and prospects and for the relevant bibliography. It is enough for our discussion to concentrate on the more relevant ones and to just keep the orders of magnitude of the experimental bounds, as shown in Table 1. Notice that electric dipole moments are displayed in units of fm rather than cm - this is kind of more natural in comparing with the anomalous magnetic moments.

2. Impact on New Physics at O(TeV) Scales

In order to evaluate to which extent these data constrain theories beyond the SM (notice the SM predictions in the table) first notice that all these transitions correspond to the same family of operators as the anomalous magnetic moment. There-

fore they can all be grouped in the expression:

$$\mu_{ij} + i d_{ij} = \frac{\Gamma_{ij}^{NP}}{M_{NP}^2} \frac{e\, m_\ell}{4\pi^2} \ell_L^i \sigma_{\mu\nu} \ell_R^j F^{\mu\nu} \qquad (1)$$

where M_{NP}^2 and Γ_{ij}^{NP} are a typical heavy mass in the radiative loops and the resulting effective coupling. The lepton mass factor, m_ℓ has been inserted because these couplings are associated to a $\Delta I = 1/2$ helicity flip and so require a Higgs v.e.v. and coupling. The $4\pi^2$ in the denominator roughly accounts for one-loop factors - barring more exotic origins for LFV&CPV. The e.m. transitions gathered in this expression are: $(g-2)_{\ell^i}$ for $i = j$, the LFV decays $\ell^j \to \ell^i \gamma$ for $j > i$, and the CPV e.d.m., d_{ℓ^i}, corresponds to $\mathrm{Im}\Gamma_{ii}^{NP}$. Let us now express M_{NP} in TeV and apply the present bounds in Table 1. This is displayed in Table 2 where the limits on M_{NP} are shown in the second column for the present data and the expected improvement in these bounds are shown in the third column. Also shown are the naive scaling factors between the same e.m. transition for different leptons as given by a simple choice of the m_ℓ.

The value of the new mass scale M_{NP} can be as low as O(200GeV) without violating the limits on $(g-2)_\mu$. On the contrary, bounds on d_e already require a suppression of two orders of magnitude in the CP violating effective coupling if $M_{NP} =$ O(TeV).

It is a well known fact that new physics is expected when experiments will go beyond the present TeV frontier in order to explain the success of the SM as a very predictive effective theory. Even on general grounds, the experimental data on LFV&CPV are already providing relevant restrictions on the flavour pattern of this new physics.

2.1. Constraints on supersymmetric models

Supersymmetry is one of the best candidates for the new physics framework and it has many sources of LFV&CPV, in particular, in the slepton mass matrices. It is worth looking in more detail for the experimental restrictions in the case of supersymmetry since we expect the sparticle masses to remain below the TeV scale to avoid

Table 1
Orders of magnitude of present experimental bounds on LFV&CPV and planned improvements

OBSERVABLE	PRESENT LIMIT	PROSPECTS	SM PREDICTION
CLFV	B.R.	B.R.	B.R.
$\tau \to \mu\,\gamma$	10^{-6}	10^{-8}	10^{-48}
$\mu \to e\,\gamma$	10^{-11}	10^{-14}	10^{-48}
EDM	e.fm	e.fm	e.fm
d_μ	10^{-5}	10^{-11}	10^{-22}
d_e	10^{-14}	10^{-16}	10^{-25}

For the detailed results and references, see Ref. [2], in these Proceedings.

Table 2
Limits on New Physics contributions on charged lepton LFV&CPV: present limits on the effective scale, expected improvement factor and relative naive relations

EXPERIMENT	M_{NP}^2 (TeV^2)	PROSPECTS	NAIVE SCALING
$(g-2)_e$	$> \Gamma_{ee}^{NP}/1000$		$\propto m_e^2$
$(g-2)_\mu$	$> \Gamma_{\mu\mu}^{NP}/20$		$\propto m_\mu^2$
$\mu \to e\,\gamma$	$> \Gamma_{\mu e}^{NP} \times 20$	$\times 30$	$\propto m_\mu$
$\tau \to \mu\,\gamma$	$> \Gamma_{\tau\mu}^{NP}/40$	$\times 10$	$\propto 0.2 m_\tau$
d_e	$> \mathrm{Im}\Gamma_{ee}^{NP} \times 70$	$\times 100$	$\propto m_e$
d_μ	$> \mathrm{Im}\Gamma_{\mu\mu}^{NP} \times 10^{-5}$	$\times 10^6$	$\propto m_\mu$

excessive fine-tuning in the model. Let us consider mSUGRA models where, by assumption, the slepton masses have no flavour structure and no relevant CP violating phases at the tree-level. We denote $\tilde{m}_L$, $\tilde{m}_R$ the masses of the scalar leptons associated to the (ℓ_L^i, ν_L^i) doublets and the ℓ_R^i singlet respectively and we skip the so-called $A-$terms for simplicity. In order to estimate the allowed deviations in the alignment in flavour space between $\tilde{m}_L$, $\tilde{m}_R$ and the charged lepton mass matrix, m_ℓ, one usually defines the ratios $\delta_{ij}^{LL} = \tilde{m}_{L,ij}^2/\tilde{m}_L^2$, $\delta_{ij}^{RR} = \tilde{m}_{R,ij}^2/\tilde{m}_R^2$, with $i \neq j$, and determine the limits on these misalignment ratios as a function of the sfermion and slepton masses from the contributions they induce in LFV decays, and in CPV e.d.m. if they are complex [3,4]. Two examples of these limits are shown in Figures 1 and 2 [4] in terms of three most relevant parameters: the slepton mass $\tilde{m}_R$, the gaugino (bino) mass $\tilde{M}_1$ and the Higgs v.e.v. angle, $\tan\beta$.

The plots show the allowed flavour misalignment δ_{12}^{LL} for sparticle masses below 1 TeV to be already $O(10^{-3})$ – it will become 30 times smaller with the planned experiments – for the case of $\mu \to e\,\gamma$, actually a stronger limit than the generic order of magnitude in Table 2 (instead the limits on δ^{RR} are much more model dependent due to destructive interference between different contributions!). This is the lepton counterpart of the supersymmetric flavour problem: the supersymmetric scalar mass generation must be endowed with such a small misalignment between slepton and lepton masses. However, since the orders of magnitude are close to those expected in radiative corrections, the alignment must be preserved by quantum contributions. This fact will be exploited in the next section.

The limits on the phases in slepton mass matrices obtained from the experimental bounds on d_e are even stronger as seen in Figure 2 for the product $\mathrm{Im}(\delta_{13}^{LL} \delta_{31}^{RR})$. Indeed, for large $\tan\beta$ their imaginary parts are at most of $O(10^{-5})$, to go down to 10^{-7} in the next future! Also notice (*cf.*
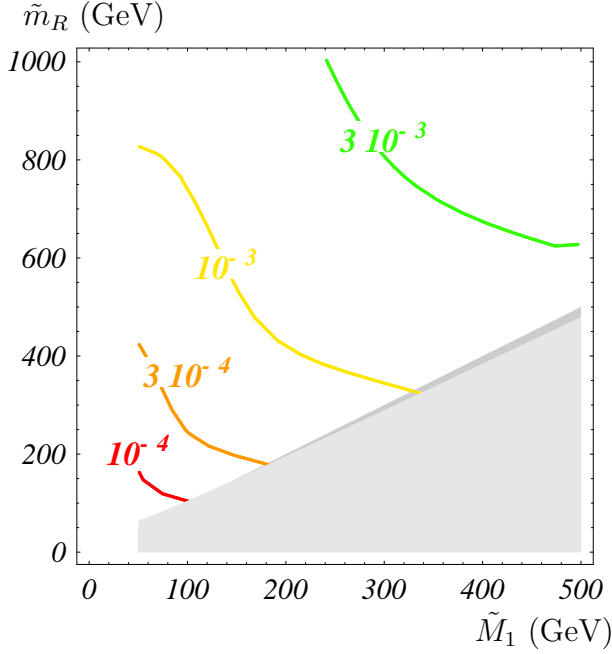
$\tilde{m}_R$ (GeV)

$\tilde{M}_1$ (GeV)

Figure 1. Upper bound on $|\delta_{12}^{LL}|$ for $\tan\beta = 10$ and the present limit $\mathrm{BR}(\mu \to e\gamma) \leq 10^{-11}$. From [4].
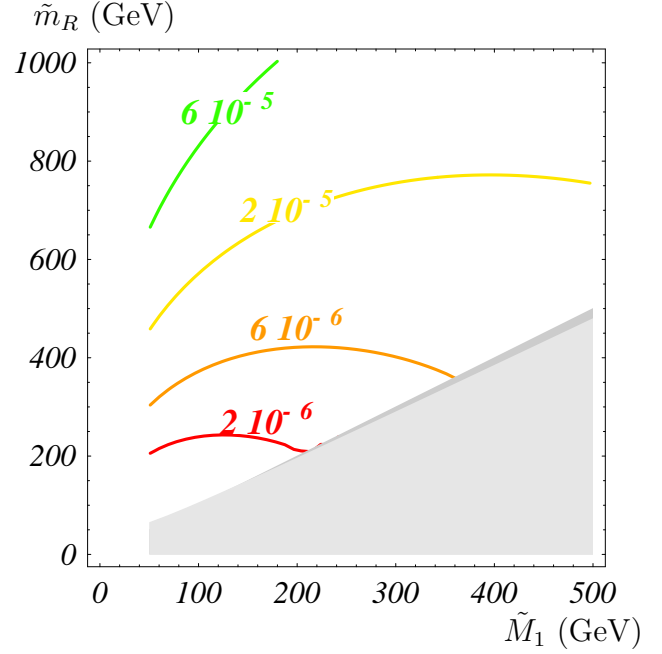
Figure 2. Upper bound on $|Im(\delta_{13}^{LL}\delta_{31}^{RR})|$ for $\tan\beta = 10$ and the present limit $d_e \leq 10^{-14}$e fm. From [4].

Table 2) that the future measurement of d_μ will provide limits comparable with the ones presently extracted from d_e. It is a remarkable fact that the e.d.m. experiments are reaching the accuracy needed to put constraints on new physics around the TeV scale at the level of radiative corrections. We now turn to discuss how this fact may tell us of properties of theories at a scale much above the TeV region, such as the seesaw models and GUT's.

3. Constraints on Seesaw and GUT's

Very heavy states that couple to sleptons leave their traces in the slepton mass matrices through their radiative corrections until they decouple from the effective supersymmetric theory. Thus, in the (type I) seesaw model, the heavy singlet neutrinos are given very large masses, M_R, and couple to the Higgs and the light leptons and

their sleptons through a Yukawa matrix Y_ν. Correspondingly, they produce a correction to the LFV part of $\tilde{m}_L$ with a misalignment $\delta_{ij}^{LL} = O(1/6)\,(Y_\nu^\dagger \ln(M_{P\ell}/M_R)\,Y_\nu)_{ij}$, where the Planck mass $M_{P\ell}$ is the cut-off for effective supergravity. This misalignment is bounded by the LFV decays [5], but only the bound on δ_{12}^{LL} is really relevant. The phases generated only by the seesaw radiative loops could produce lepton e.d.m. at observable rates only if the M_R eigenvalues are strongly hierarchical [6,7]. On the other hand, in the simplest versions of SU(5) GUT's, the colour triplet partners of the Higgs bosons, must get large masses, M_T, through the doublet-triplet mechanism, while they couple to the right-handed leptons through a Yukawa matrix which is approximately Y_u, the usual couplings of the up quarks. Correspondingly, radiative corrections are generated to $\delta_{ij}^{RR} = O(1/6)\,(Y_u^\dagger \ln(M_{P\ell}/M_T)\,Y_u)_{ij}$.

They induce and are constrained by LFV decays as well [8]. What about CPV corrections to the slepton masses from these heavy neutrinos and colour triplets? It has been recently shown [9,7] that with SU(5) GUT $\oplus$ seesaw one obtains interesting constraints on Y_ν matrix elements from the d_e experiments, This is shown in Figure 3 [7] for $(Y_\nu^\dagger Y_\nu)_{13}$, where ϕ_{td} is the phase in the quark CKM matrix. Remember that these limits will be improved by two orders of magnitude.
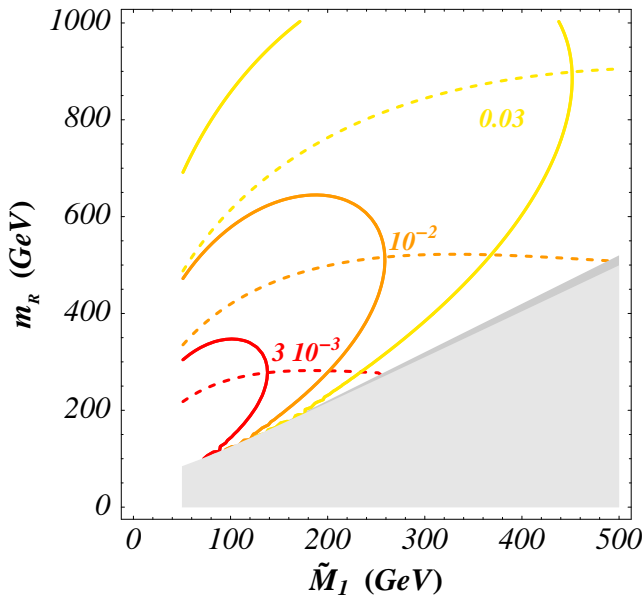

Figure 3. Upper limit on $|Im(e^{-i\phi_{td}}(Y_\nu^\dagger Y_\nu)_{13})|$ from the present bound on d_e, for $\tan\beta = 10$ and a triplet mass $M_T = 2 \times 10^{16}$ GeV, $A_0^2 = 2m_0^2$ (solid line), $A_0^2 = \tilde{M}_{1/2}^2$ (dashed line). From [7].

Another way to see the relevance of the restrictions from d_e on new physics is to show that they are competitive with the well-known constraints on the colour triplets masses from proton life-time, τ_p [10]. Let us take for definiteness $M_R \simeq M_T \simeq 10^{17}$GeV and "typical" sparticle masses, $\tilde{m}_R = 2\tilde{M}_1 = 400$GeV and, e.g.,

$\tan\beta = 3$, with mSUGRA mass conditions and let us calculate d_e and τ_p in the simplest models (which are not meant to be more theoretically justified, see e.g. [11]): (I) SU(5) with seesaw and the simplifying assumption, $Y_\nu \approx Y_u$; (II) SO(10) with two colour triplets and $M_{T_1} \approx M_{T_2}$; (III) SO(10) with pseudo-Dirac like colour triplets, $(r + 1)M_{T_1} = (r - 1)M_{T_2}$ with $r \ll 1$. The results are summarized in Table 3. While models I and II predict τ_p much below the experimental value, the pseudo-Dirac trick introduces large negative interference between the two massive colour triplets increasing τ_p by almost two orders of magnitude (depending on the various Yukawa phases; note that the quark CKM phase in Y_u alone is enough unless it is compensated by the unknown phases). But the supersymmetric SO(10) models violate the limits on d_e, at least in the simpler versions. While τ_p could be reduced by interference, the different contributions add up in the wave function renormalisation that leads to d_e.

4. Constraints on Abelian Flavour Models

In order to explicitly see the impact of the $\mu \to e\,\gamma$ and d_e experiments on flavour models to explain the fermion masses, including the neutrinos, let us consider the Froggatt-Nielsen simplest model with an $U(1)$ flavour group [12]. It is assumed to be broken by field whose v.e.v. defines a small number ϵ with respect to the cut-off scale of the model. Charges are associated to each lepton field as follows: ℓ_i to the doublets $(e, \nu)_i$, e_i to e_i^c, n_i to ν_i^c, with $i = 1, 2, 3$ as the flavour index, $h_{1,2}$ to the two Higgses. In this way one obtains, after the $U(1)$ breaking, effective Yukawa couplings endowed with some kind of hierarchy in the eigenvalues and the mixings in terms of powers of ϵ defined by the flavour charges. Ordering the heavy right-handed neutrinos by $M_1 \le M_2 \le M_3$, the present neutrino oscillation data (mass differences and mixing angles for atmospheric and solar neutrinos) basically fix the charges: $\ell_3 = \ell_2 = \ell_1 - 1$, together with $\epsilon = O(1/6)$, close to the Cabibbo angle that plays a similar role in the construction of quark mass models. The masses M_i remain arbitrary, but leptogenesis is possible only

Table 3
Electron e.d.m. versus proton lifetime as tests of GUT's. From [10].

	$\tau(p \to K^+\nu)/\ (10^{33}\text{yrs})$	$d_e/(10^{-14}\text{e.fm})$
EXPERIMENTAL BOUNDS	> 2	< 1
(I) SU(5) w/ $Y_\nu = Y_u$	$O(0.1)$	0.2
(II) SO(10) w/ $M_{T_1} = M_{T_2}$	$O(0.1)$	2.0
(III) SO(10) w/ $M_{T_1} \approx -M_{T_2}$	$0.2 \leftrightarrow 7$	$1.8 \leftrightarrow 2.0$

if $M_1 = O(10^{11}\text{GeV})$. Because the ℓ_i are now given, the present experimental limit on $\mu \to e\,\gamma$ requires $M_3 < O(10^{13}\text{GeV})$, for $\tan\beta = 10$. The charges n_i are now more or less fixed so that the orders of magnitude of all other LFV&CPV predictions, in particular d_e, come out much below the present experiments. With the expected improvement by a factor 30 in B.R.$(\mu \to e\,\gamma)$, one would get $M_3 < O(10^{11}\text{GeV})$ and this very popular model is close to be excluded. What is nice in this exercise is that the constraints from different observables apply to different parameters and complement each other in an explicit way.

5. Conclusions

The LFV&CPV experiments, mainly $\mu \to e\,\gamma$ and d_e – but hopefully also d_μ in the near future – are a selective tool for New Physics model building. They provide indirect tests which are already sensitive to physics in the TeV region at the level of accuracy of the radiative corrections. They are pure signals of new phenomena around the TeV scale since the SM contributions are definitely negligible. It is quite remarkable that they also give effective tests of the LFV&CPV in seesaw couplings and in grand-unified theories. In particular, the limits on d_e nicely complement the proton decay bounds in selecting O(10) models!

The devised improvements in these experiments are important and theoretically welcome. The current direct measurement of d_μ, improving the present indirect bound, motivates a simple question [13]: to what extent can the naive relation $d_\mu/d_e \simeq m_\mu/m_e$ be violated? Needless to say, the fact that they are getting into the level of accuracy corresponding to where we do expect the new physics to show up means they are on the way to discover LFV&CPV in charged lepton transitions.

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